SpecFun
A Special Functions Library for the HP48 with ALG48 Version 4.0

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1 Acknowledgements, copyright & disclaimer of warranty

Special thanks to Dr. Mark A. Ordal whose Bessel function implementations were the basis for the ones in `SpecFun`.

2 Overview

`SpecFun` defines a number of commonly used "special functions" not provided by the `HP48`, including Bessel functions, error functions, Gamma and Beta functions, and polynomial generating functions. Polynomial generating functions include Legendre, Hermite, Tschebyscheff and Laguerre polynomials as well as spherical harmonics.

`SpecFun` works in conjunction with the `ALG48` library © by Claude-Nicolas Fiechter and Mika Heiskanen and needs `ALG48` to be installed to work properly. The function names in `SpecFun` were chosen so as to be displayed in the customary way when using the `EQSTK` library © by the same authors.

3 Installation

`SpecFun` takes approximately 6Kb of memory and works in both `HP48G(X)` and `HP48S(X)`. `SpecFun` uses some of the internal subroutines in `ALG48`, and thus requires it to be installed. See the `ALG48` documentation for information on installing `ALG48`.

Due to the special optimization features used in `ALG48`, not all storage combinations are allowed. The following ones are possible:

- In `SX` there are no restrictions
- In `GX` if `ALG48` is in port 0 or port 1 then `SpecFun` can be stored in any port.
- In `GX` if `ALG48` is stored in port 2 (or higher) then `SpecFun` must be stored in the same port, or port 0.

Installing `ALG48` and `SpecFun` in the same port seems the most natural choice and is highly recommended.

`SpecFun` is an auto-attaching library (library number 911). To install it on your `HP48` download the file `specfun.lib` onto your calculator (in *binary* mode), put the content of the created variable on the stack, store it the port of your choice (e.g., `'SPECFUN.LIB' DUP RCL SWAP PURGE 0 STO`) and power-cycle the calculator.

4 Use

The functions defined by `SpecFun` are divided in two groups: special functions and polynomial generating functions. We give below the definitions of all the functions implemented in `SpecFun`. Section 4.3 gives some example of utilization.

4.1 Special Functions

The special functions accept all combinations of real and symbolic arguments, but are evaluated only for real arguments. To evaluate the Bessel functions the order argument n has to be an integer value, otherwise an `"Undefined Result"` error is generated.

- Gamma Function

$$\texttt{GAMMA}(x) = \int_0^\infty e^{-t} t^{x-1}\, dt \quad , \quad x > 0$$

- Beta Function

$$\texttt{BETA}(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt = \frac{\texttt{GAMMA}(x)\texttt{GAMMA}(y)}{\texttt{GAMMA}(x+y)} \quad , \quad x, y > 0$$

- Error Functions

$$\begin{aligned} \texttt{ERF}(x) &= \frac{2}{\pi}\int_0^x e^{-t^2}\, dt \\ \texttt{ERFC}(x) &= \frac{2}{\pi}\int_x^\infty e^{-t^2}\, dt = 1 - \texttt{ERF}(x) \end{aligned}$$

- Bessel Functions

$$\begin{aligned} \texttt{J.n}(n,x) &= J_n(x) &&= \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{n+2r}}{r! \cdot \texttt{GAMMA}(n+r+1)} \\ \texttt{Y.n}(n,x) &= Y_n(x) &&= \lim_{p\to n} \frac{J_p(x)\cos(p\pi) - J_{-p}(x)}{\sin(p\pi)} \quad , \quad x \geq 0 \\ \texttt{I.n}(n,x) &= I_n(x) &&= i^{-n} J_n(ix) \\ \texttt{K.n}(n,x) &= K_n(x) &&= \lim_{p\to n} \frac{I_{-p}(x) - I_p(x)}{\sin(p\pi)} \cdot \frac{\pi}{2} \quad , \quad x \geq 0 \end{aligned}$$

- Quick access commands

$$\begin{aligned} \texttt{J.0}(x) &= J_0(x) \\ \texttt{J.1}(x) &= J_1(x) \end{aligned}$$

4.2 Polynomial Generating Functions

The polynomial generating functions expect the order arguments (n,m) to be positive integer values and the variables (x,a,b) to be symbolic or identifier objects. A simplified output form is used for the Legendre polynomials when the variable argument is of the form "cos(x)", for some arbitrary sub-expression x (see Section 4.3).

- Legendre Polynomials

$$\begin{aligned} \texttt{P.n}(n,x) &= P_n(x) = \frac{1}{2^n n!} \cdot \frac{d^n}{dx^n}\left(x^2-1\right)^n \\ \texttt{P.nm}(n,m,x) &= P_n^m(x) = \left(1-x^2\right)^{m/2} \cdot \frac{d^m}{dx^m} P_n(x) \end{aligned}$$

- Spherical Harmonics

$$\texttt{Y.nm}(n,m,a,b) = Y_n^m(a,b) = (-1)^m \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^m(\cos a) e^{imb}$$

- Hermite Polynomials

$$\texttt{H.n}(n,x) = H_n(x) = (-1)^n \cdot e^{x^2/2} \cdot \frac{d^n}{dx^n} e^{-x^2/2}$$

- Tschebyscheff Polynomials

$$\begin{aligned} \texttt{T.n}(n,x) &= T_n(x) = \cos\left(n \cdot \arccos(x)\right) \\ \texttt{U.n}(n,x) &= U_n(x) = \frac{\sin\left((n+1)\arccos(x)\right)}{\sqrt{1-x^2}} \end{aligned}$$

- Laguerre Polynomials

$$\begin{aligned} \texttt{L.n}(x) &= L_n(x) = \frac{e^x}{n!} \cdot \frac{d^n}{dx^n}\left(x^n e^{-x}\right) \\ \texttt{L.nm}(x) &= L_n^m(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x) \end{aligned}$$

4.3 Examples

`5 X P.n` $\Rightarrow$	$\frac{1}{8}(63X^5 - 70X^3 + 15X)$	Standard output form
`2 1 X P.nm` $\Rightarrow$	$3X\left(1-X^2\right)^{1/2}$	Standard output form
`2 1 COS(X) P.nm` $\Rightarrow$	$3\cos(X)\sin(X)$	Special output for "cos(x)"
`2 1 t p Y.nm` $\Rightarrow$	$-\sqrt{\frac{5}{24\pi}} \cdot 3\cos(t)\sin(t)e^{ip}$	Usually $t, p = \theta, \phi$
`5 COS(X) T.n` $\Rightarrow$	$16\cos(x)^5 - 20\cos(x)^3 + 5\cos(x)$	Expansion of $\cos(5x)$

5 Changes from Previous Versions

5.1 Changes from version 3.0 to 4.0

- Fixed Gamma function for arguments smaller than -8

5.2 Changes from version 2.4 to 3.0

- Removed infinite precision factorial command.
- Extended Bessel function ranges to $x < 0$ and $n < 0$ when possible.
- Allowed $m > n$ for polynomial generating functions.

6 Contact

Gifts `:)`, bug reports, and constructive comments and suggestions can be sent to either one of the following addresses.

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